



Lesson 4: Numbers Raised to the Zeroth Power

Student Outcomes

- Students know that a number raised to the zeroth power is equal to one.
- Students recognize the need for the definition to preserve the properties of exponents.

Classwork

Concept Development (5 minutes)

Let us summarize our main conclusions about exponents. For any numbers x , y and any positive integers m , n , the following holds:

$$x^m \cdot x^n = x^{m+n} \quad (1)$$

$$(x^m)^n = x^{mn} \quad (2)$$

$$(xy)^n = x^n y^n \quad (3)$$

For any numbers x , y , and any positive integers m , n , the following holds:

$$x^m \cdot x^n = x^{m+n} \quad (1)$$

$$(x^m)^n = x^{mn} \quad (2)$$

$$(xy)^n = x^n y^n \quad (3)$$

Definition: _____

And if we assume $x > 0$ in equation (4) and $y > 0$ in equation (5) below, then we also have:

$$\frac{x^m}{x^n} = x^{m-n}, \quad m > n \quad (4)$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \quad (5)$$

There is an obvious reason why the x in (4) and the y in (5) must be nonzero: we cannot divide by 0. However, the reason for further restricting x and y to be positive is only given when fractional exponents have been defined. This will be done in high school.

We group equations (1)–(3) together because they are the foundation on which all the results about exponents rest. When they are suitably generalized as will be done below, they will imply (4) and (5). Therefore, we concentrate on (1)–(3).



The most important feature of (1)–(3) is that they are *simple* and they are *formally (symbolically) natural*. Mathematicians want these three identities to continue to hold for *all* exponents m and n , without the restriction that m and n be positive integers because of these two desirable qualities. We will have to do it one step at a time. Our goal in this grade is to extend the validity of (1)–(3) to all *integers* m and n .

Exploratory Challenge (10 minutes)

The first step in this direction is to introduce the definition of the 0^{th} exponent of a positive number and then use it to prove that (1)–(3) remain valid when m and n are not just positive integers, but all whole numbers (including 0). Since our goal is to make sure (1)–(3) remain valid even when m and n may be 0, the very definition of the 0^{th} exponent of a number must pose no *obvious* contradiction to (1)–(3). With this in mind, let us consider what it means to raise a positive number x to the zeroth power. For example, what should 3^0 mean?

- *Students will likely respond that 3^0 should equal 0.* When they do, demonstrate why that would contradict our existing understanding of properties of exponents using (1). Specifically, if m is a positive integer and we let $3^0 = 0$, then

$$3^m \cdot 3^0 = 3^{m+0},$$

but since we let $3^0 = 0$ it means that the left side of the equation would equal zero. That creates a contradiction because

$$0 \neq 3^{m+0}$$

Therefore, letting $3^0 = 0$ will not help us to extend (1)–(3) to all whole numbers m and n .

- *Next, students may say that we should let $3^0 = 3$.* Show the two problematic issues this would lead to. First, we have already learned that by definition $x^1 = x$ in Lesson 1, and we do not want to have two powers that yield the same result. Second, it would violate the existing rules we have developed: Looking specifically at (1) again, if we let $3^0 = 3$, then

$$3^m \cdot 3^0 = 3^{m+0},$$

but

$$\begin{aligned} 3^m \cdot 3^0 &= \underbrace{3 \times \dots \times 3}_{m \text{ times}} \cdot 3 \\ &= 3^{m+1} \end{aligned}$$

which is a contradiction again.

If we believe that equation (1) should hold even when (let us say) $n = 0$, then for example:

$3^{2+0} = 3^2 \times 3^0$, which is the same as $3^2 = 3^2 \times 3^0$, and therefore after multiplying both sides by the number $\frac{1}{3^2}$ we get $1 = 3^0$. In the same way, our belief that (1) should hold when either m or n is 0 would lead us to the conclusion that we should define $x^0 = 1$ for any nonzero x . Therefore, we give the following definition:

MP.6 Definition: For any positive number x , we define $x^0 = 1$.

Students will need to write this definition of x^0 in the lesson summary box on their classwork paper.

Exploratory Challenge 2 (10 minutes)

Now that x^n is defined for all whole numbers n , check carefully that (1)–(3) remain valid for all whole numbers m and n .

MP.3

Have students independently complete Exercise 1, provide correct values for m and n before proceeding. (Development of cases (A)–(C)).

Exercise 1

List all possible cases of whole numbers m and n for identity (1). More precisely, when $m > 0$ and $n > 0$, we already know that (1) is correct. What are the other possible cases of m and n for which (1) is yet to be verified?

Case (A): $m > 0$ and $n = 0$

Case (B): $m = 0$ and $n > 0$

Case (C): $m = n = 0$

Model how to check the validity of a statement using Case (A) with equation (1) as part of Exercise 2. Have students work independently or in pairs to check the validity of (1) in Case (B) and Case (C) to complete Exercise 2. Next, have students check the validity of equations (2) and (3) using Cases (A)–(C) for Exercises 3 and 4.

Exercise 2

Check that equation (1) is correct for each of the cases listed in Exercise 1.

Case (A): $x^m \cdot x^0 = x^m$? Yes, because $x^m \cdot x^0 = x^m \cdot 1 = x^m$.

Case (B): $x^0 \cdot x^n = x^n$? Yes, because $x^0 \cdot x^n = 1 \cdot x^n = x^n$.

Case (C): $x^0 \cdot x^0 = x^0$? Yes, because $x^0 \cdot x^0 = 1 \cdot 1 = x^0$.

Exercise 3

Do the same with equation (2) by checking it case-by-case.

Case (A): $(x^m)^0 = x^{0 \times m}$? Yes, because x^m is a number and a number raised to a zero power is 1. $1 = x^0 = x^{0 \times m}$.

So, the left side is 1. The right side is also 1 because $x^{0 \times m} = x^0 = 1$.

Case (B): $(x^0)^n = x^{n \times 0}$? Yes, because by definition $x^0 = 1$ and $1^n = 1$, so the left side is equal to 1. The right side is equal to $x^0 = 1$, and so both sides are equal.

Case (C): $(x^0)^0 = x^{0 \times 0}$? Yes, because by definition of the zeroth power of x , both sides are equal to 1.

Exercise 4

Do the same with equation (3) by checking it case-by-case.

Case (A): $(xy)^0 = x^0 y^0$? Yes, because the left side is 1 by the definition of the zeroth power while the right side is $1 \times 1 = 1$.

Case (B): Since $n > 0$, we already know that (3) is valid.

Case (C): This is the same as Case (A) which we have already shown to be valid.

Exploratory Challenge 3 (5 minutes)

Students will practice writing numbers in expanded notation in Exercises 5 and 6. Students will use the definition of x^0 , for any positive number x , learned in this lesson.

Clearly state that you want to see the ones digit multiplied by 10^0 . That is the important part of the expanded notation. This will lead to the use of negative powers of 10 for decimals in Lesson 5.

Scaffolding:

- You may need to remind students how to write numbers in expanded form with Exercise 5.

Exercise 5

Write the expanded form of 8,374 using the exponential notation.

$$8,374 = (8 \times 10^3) + (3 \times 10^2) + (7 \times 10^1) + (4 \times 10^0)$$

Exercise 6

Write the expanded form of 6,985,062 using the exponential notation.

$$6,985,062 = (6 \times 10^6) + (9 \times 10^5) + (8 \times 10^4) + (5 \times 10^3) + (0 \times 10^2) + (6 \times 10^1) + (2 \times 10^0)$$

Summary Discussion (3 minutes)

Summarize, or have students summarize, the lesson.

- The rules of exponents that students have worked on prior to today only work for positive integer exponents, now those same rules have been extended to all whole numbers.
- The next logical step is to attempt to extend these rules to all integer exponents.

Exit Ticket (2 minutes)**Fluency Activity (10 minutes)**

Sprint: Rewrite expressions with the same base for positive exponents only. Make sure to tell the students that all letters within the problems of the sprint are meant to denote numbers. This activity can be administered at any point throughout the lesson.



Name _____

Date _____

Lesson 4: Numbers Raised to the Zeroth Power

Exit Ticket

1. Simplify the following expression as much as possible.

$$\frac{4^{10}}{4^{10}} \cdot 7^0 =$$

2. Let a and b be two numbers. Use the distributive law and the definition of zeroth power to show that the numbers $(a^0 + b^0)a^0$ and $(a^0 + b^0)b^0$ are equal.

Exit Ticket Sample Solutions

1. Simplify the following expression as much as possible.

$$\frac{4^{10}}{4^{10}} \cdot 7^0 = 4^{10-10} \cdot 1 = 4^0 \cdot 1 = 1 \cdot 1 = 1$$

2. Let a and b be two numbers. Use the distributive law and the definition of zeroth power to show that the numbers $(a^0 + b^0)a^0$ and $(a^0 + b^0)b^0$ are equal.

$$\begin{aligned} (a^0 + b^0)a^0 &= a^0 \cdot a^0 + b^0 \cdot a^0 \\ &= a^{0+0} + a^0b^0 \\ &= a^0 + a^0b^0 \\ &= 1 + 1 \cdot 1 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

$$\begin{aligned} (a^0 + b^0)b^0 &= a^0 \cdot b^0 + b^0 \cdot b^0 \\ &= a^0b^0 + b^{0+0} \\ &= a^0b^0 + b^0 \\ &= 1 \cdot 1 + 1 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

Since both numbers are equal to 2, they are equal.

Problem Set Sample Solutions

Let x, y be numbers ($x, y \neq 0$). Simplify each of the following expressions of numbers.

1.

$$\begin{aligned} \frac{y^{12}}{y^{12}} &= y^{12-12} \\ &= y^0 \\ &= 1 \end{aligned}$$

2.

$$\begin{aligned} 9^{15} \cdot \frac{1}{9^{15}} &= \frac{9^{15}}{9^{15}} \\ &= 9^{15-15} \\ &= 9^0 \\ &= 1 \end{aligned}$$

3.

$$\begin{aligned} (7(123456.789)^4)^0 &= \\ &= 7^0(123456.789)^{4 \cdot 0} \\ &= 7^0(123456.789)^0 \\ &= 1 \end{aligned}$$

4.

$$\begin{aligned} 2^2 \cdot \frac{1}{2^5} \cdot 2^5 \cdot \frac{1}{2^2} &= \frac{2^2 \cdot 2^5}{2^2 \cdot 2^5} \\ &= 2^{2-2} \cdot 2^{5-5} \\ &= 2^0 \cdot 2^0 \\ &= 1 \end{aligned}$$

5.

$$\begin{aligned} \frac{x^{41}}{y^{15}} \cdot \frac{y^{15}}{x^{41}} &= \frac{x^{41} \cdot y^{15}}{y^{15} \cdot x^{41}} \\ &= \frac{x^{41}}{x^{41}} \cdot \frac{y^{15}}{y^{15}} \\ &= x^{41-41} \cdot y^{15-15} \\ &= x^0 \cdot y^0 \\ &= 1 \end{aligned}$$



Sprint 1: Rewrite each item as an equivalent expression in exponential notation. All letters denote numbers.

1.	$2^2 \cdot 2^3 =$
2.	$2^2 \cdot 2^4 =$
3.	$2^2 \cdot 2^5 =$
4.	$3^7 \cdot 3^1 =$
5.	$3^8 \cdot 3^1 =$
6.	$3^9 \cdot 3^1 =$
7.	$7^6 \cdot 7^2 =$
8.	$7^6 \cdot 7^3 =$
9.	$7^6 \cdot 7^4 =$
10.	$11^{15} \cdot 11 =$
11.	$11^{16} \cdot 11 =$
12.	$2^{12} \cdot 2^2 =$
13.	$2^{12} \cdot 2^4 =$
14.	$2^{12} \cdot 2^6 =$
15.	$99^5 \cdot 99^2 =$
16.	$99^6 \cdot 99^3 =$
17.	$99^7 \cdot 99^4 =$
18.	$5^8 \cdot 5^2 =$
19.	$6^8 \cdot 6^2 =$
20.	$7^8 \cdot 7^2 =$
21.	$r^8 \cdot r^2 =$
22.	$s^8 \cdot s^2 =$

23.	$6^3 \cdot 6^2 =$
24.	$6^2 \cdot 6^3 =$
25.	$(-8)^3 \cdot (-8)^7 =$
26.	$(-8)^7 \cdot (-8)^3 =$
27.	$(0.2)^3 \cdot (0.2)^7 =$
28.	$(0.2)^7 \cdot (0.2)^3 =$
29.	$(-2)^{12} \cdot (-2)^1 =$
30.	$(-2.7)^{12} \cdot (-2.7)^1 =$
31.	$1.1^6 \cdot 1.1^9 =$
32.	$57^6 \cdot 57^9 =$
33.	$x^6 \cdot x^9 =$
34.	$2^8 \cdot 4 =$
35.	$2^8 \cdot 4^2 =$
36.	$2^8 \cdot 16 =$
37.	$16 \cdot 4^3 =$
38.	$3^2 \cdot 9 =$
39.	$3^2 \cdot 27 =$
40.	$3^2 \cdot 81 =$
41.	$5^4 \cdot 25 =$
42.	$5^4 \cdot 125 =$
43.	$8 \cdot 2^{10} =$
44.	$16 \cdot 2^{10} =$



Sprint 2: Rewrite each item as an equivalent expression in exponential notation. All letters denote numbers.

1.	$5^2 \cdot 5^3 =$
2.	$5^2 \cdot 5^4 =$
3.	$5^2 \cdot 5^5 =$
4.	$2^7 \cdot 2^1 =$
5.	$2^8 \cdot 2^1 =$
6.	$2^9 \cdot 2^1 =$
7.	$4^6 \cdot 4^2 =$
8.	$4^6 \cdot 4^3 =$
9.	$4^6 \cdot 4^4 =$
10.	$8^{15} \cdot 8 =$
11.	$8^{16} \cdot 8 =$
12.	$9^{12} \cdot 9^2 =$
13.	$9^{12} \cdot 9^4 =$
14.	$9^{12} \cdot 9^6 =$
15.	$23^5 \cdot 23^2 =$
16.	$23^6 \cdot 23^3 =$
17.	$23^7 \cdot 23^4 =$
18.	$14^7 \cdot 14^3 =$
19.	$15^7 \cdot 15^3 =$
20.	$16^7 \cdot 16^3 =$
21.	$x^7 \cdot x^3 =$
22.	$y^7 \cdot y^3 =$

23.	$7^3 \cdot 7^2 =$
24.	$7^2 \cdot 7^3 =$
25.	$(-4)^3 \cdot (-4)^{11} =$
26.	$(-4)^{11} \cdot (-4)^3 =$
27.	$(0.2)^3 \cdot (0.2)^{11} =$
28.	$(0.2)^{11} \cdot (0.2)^3 =$
29.	$(-2)^9 \cdot (-2)^5 =$
30.	$(-2.7)^5 \cdot (-2.7)^9 =$
31.	$3.1^6 \cdot 3.1^6 =$
32.	$57^6 \cdot 57^6 =$
33.	$z^6 \cdot z^6 =$
34.	$4 \cdot 2^8 =$
35.	$4^2 \cdot 2^8 =$
36.	$16 \cdot 2^8 =$
37.	$16 \cdot 4^2 =$
38.	$9 \cdot 3^2 =$
39.	$3^3 \cdot 9 =$
40.	$3^3 \cdot 27 =$
41.	$5^6 \cdot 25 =$
42.	$5^6 \cdot 125 =$
43.	$2^{10} \cdot 4 =$
44.	$2^{10} \cdot 16 =$

Sprint 1 Solutions

Rewrite each item as an equivalent expression in exponential notation. All letters denote numbers.

1.	$2^2 \cdot 2^3 = 2^5$
2.	$2^2 \cdot 2^4 = 2^6$
3.	$2^2 \cdot 2^5 = 2^7$
4.	$3^7 \cdot 3^1 = 3^8$
5.	$3^8 \cdot 3^1 = 3^9$
6.	$3^9 \cdot 3^1 = 3^{10}$
7.	$7^6 \cdot 7^2 = 7^8$
8.	$7^6 \cdot 7^3 = 7^9$
9.	$7^6 \cdot 7^4 = 7^{10}$
10.	$11^{15} \cdot 11 = 11^{16}$
11.	$11^{16} \cdot 11 = 11^{17}$
12.	$2^{12} \cdot 2^2 = 2^{14}$
13.	$2^{12} \cdot 2^4 = 2^{16}$
14.	$2^{12} \cdot 2^6 = 2^{18}$
15.	$99^5 \cdot 99^2 = 99^7$
16.	$99^6 \cdot 99^3 = 99^9$
17.	$99^7 \cdot 99^4 = 99^{11}$
18.	$5^8 \cdot 5^2 = 5^{10}$
19.	$6^8 \cdot 6^2 = 6^{10}$
20.	$7^8 \cdot 7^2 = 7^{10}$
21.	$r^8 \cdot r^2 = r^{10}$
22.	$s^8 \cdot s^2 = s^{10}$

23.	$6^3 \cdot 6^2 = 6^5$
24.	$6^2 \cdot 6^3 = 6^5$
25.	$(-8)^3 \cdot (-8)^7 = (-8)^{10}$
26.	$(-8)^7 \cdot (-8)^3 = (-8)^{10}$
27.	$(0.2)^3 \cdot (0.2)^7 = (0.2)^{10}$
28.	$(0.2)^7 \cdot (0.2)^3 = (0.2)^{10}$
29.	$(-2)^{12} \cdot (-2)^1 = (-2)^{13}$
30.	$(-2.7)^{12} \cdot (-2.7)^1 = (-2.7)^{13}$
31.	$1.1^6 \cdot 1.1^9 = 1.1^{15}$
32.	$57^6 \cdot 57^9 = 57^{15}$
33.	$x^6 \cdot x^9 = x^{15}$
34.	$2^8 \cdot 4 = 2^{10}$
35.	$2^8 \cdot 4^2 = 2^{12}$
36.	$2^8 \cdot 16 = 2^{12}$
37.	$16 \cdot 4^3 = 4^5$
38.	$3^2 \cdot 9 = 3^4$
39.	$3^2 \cdot 27 = 3^5$
40.	$3^2 \cdot 81 = 3^6$
41.	$5^4 \cdot 25 = 5^6$
42.	$5^4 \cdot 125 = 5^7$
43.	$8 \cdot 2^{10} = 2^{13}$
44.	$16 \cdot 2^{10} = 2^{14}$

Sprint 2 Solutions

Rewrite each item as an equivalent expression in exponential notation. All letters denote numbers.

1.	$5^2 \cdot 5^3 = 5^5$
2.	$5^2 \cdot 5^4 = 5^6$
3.	$5^2 \cdot 5^5 = 5^7$
4.	$2^7 \cdot 2^1 = 2^8$
5.	$2^8 \cdot 2^1 = 2^9$
6.	$2^9 \cdot 2^1 = 2^{10}$
7.	$4^6 \cdot 4^2 = 4^8$
8.	$4^6 \cdot 4^3 = 4^9$
9.	$4^6 \cdot 4^4 = 4^{10}$
10.	$8^{15} \cdot 8 = 8^{16}$
11.	$8^{16} \cdot 8 = 8^{17}$
12.	$9^{12} \cdot 9^2 = 9^{14}$
13.	$9^{12} \cdot 9^4 = 9^{16}$
14.	$9^{12} \cdot 9^6 = 9^{18}$
15.	$23^5 \cdot 23^2 = 23^7$
16.	$23^6 \cdot 23^3 = 23^9$
17.	$23^7 \cdot 23^4 = 23^{11}$
18.	$14^7 \cdot 14^3 = 14^{10}$
19.	$15^7 \cdot 15^3 = 15^{10}$
20.	$16^7 \cdot 16^3 = 16^{10}$
21.	$x^7 \cdot x^3 = x^{10}$
22.	$y^7 \cdot y^3 = y^{10}$

23.	$7^3 \cdot 7^2 = 7^5$
24.	$7^2 \cdot 7^3 = 7^5$
25.	$(-4)^3 \cdot (-4)^{11} = (-4)^{14}$
26.	$(-4)^{11} \cdot (-4)^3 = (-4)^{14}$
27.	$(0.2)^3 \cdot (0.2)^{11} = (0.2)^{14}$
28.	$(0.2)^{11} \cdot (0.2)^3 = (0.2)^{14}$
29.	$(-2)^9 \cdot (-2)^5 = (-2)^{14}$
30.	$(-2.7)^5 \cdot (-2.7)^9 = (-2.7)^{14}$
31.	$3.1^6 \cdot 3.1^6 = 3.1^{12}$
32.	$57^6 \cdot 57^6 = 57^{12}$
33.	$z^6 \cdot z^6 = z^{12}$
34.	$4 \cdot 2^8 = 2^{10}$
35.	$4^2 \cdot 2^8 = 2^{12}$
36.	$16 \cdot 2^8 = 2^{12}$
37.	$16 \cdot 4^2 = 4^4$
38.	$9 \cdot 3^2 = 3^4$
39.	$3^3 \cdot 9 = 3^5$
40.	$3^3 \cdot 27 = 3^6$
41.	$5^6 \cdot 25 = 5^8$
42.	$5^6 \cdot 125 = 5^9$
43.	$2^{10} \cdot 4 = 2^{12}$
44.	$2^{10} \cdot 16 = 2^{14}$